Big Bang in AdS_5 with external field and flat 4d Universe

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Abstract

We describe spontaneous creation of the Brane World in AdS_5 with external field. The resulting Brane World consists of a flat 4d spatially finite expanding Universe and curved expanding "regulator" branes. No negative tension branes are involved.

1. Recently a lot of attention has been paid to the possibility to have large [1] and even infinite [2] extra dimensions instead of traditional Kaluza-Klein picture where extra dimensions are extremely small and thus invisible at moderate energy scales.

The idea of those considerations is that matter [3], gauge fields [4] and gravity [2] can be a sort of localized on branes (in the present context branes can be viewed on as fundamental branes or effective domain walls) so that, say, Newton law is the same as in flat 4d world with small corrections beyond the reach of the current experimental physics.

A particular construction in [2] included two slices of AdS_5 matched with each other along two 3-branes (i.e. branes with 4d world-sheet). The branes were localized on a circle in 5th dimension, radius of the circle being arbitrarily large. The slices of AdS_5 were taken in such a way that the 3-branes were flat in either of them. Main feature of this model is that when parameters of the model (cosmological constant in AdS_5 and tension of the brane) are tuned to guaranty the existence of the stable brane located along the flat section

of AdS, one automatically obtains zero cosmological constant in 4d world on the brane. A disadvantage of the model (as well as of its modifications, see e.g. modifications with three branes, two of which have negative tensions [5] and some others [6]) is the necessity to have auxiliary brane(s) with negative tension. The negative tension branes (regulator branes, R-branes) have been realized as a crucial problem of the construction (see, e.g. [7]), and attempts have been taken to overcome it [8] or even to abandon the Brane World in favor of the other ideas about vanishing cosmological constant, e.g. the one of [9] which was recently revisited in [10].

In a recent paper [11] the present authors have suggested a tunneling mechanism resulting in a brane configuration looking very similar to the configurations in [2], [5]. The R-branes in the configuration produced are not parallel to the physical brane (RS-brane) and rapidly expand. RS-brane is spatially finite, the size being of the same order as the distance to R-brane. What is important that none of the branes has negative tension. A peculiar feature of the configuration is that RS-brane is not located along the flat section of AdS, hence the 4d cosmological constant is not zero, and even 4d localization of gravity on RS-brane, perhaps, requires additional justification.

In the present letter we describe a tunneling into the Brane World such that the resulting RS-brane is a peace of the same flat section of AdS_5 as in [2]. Thus our 4d Universe living on RS-brane is flat and spatially finite and we can take for granted from [2] the 4d localization of gravity on the RS-brane and the validity of 4d Newton law far enough in the future. RS-brane is restricted by junction manifold where RS-brane meets with rapidly expanding R-branes.

The setup of our consideration is as follows. Originally, one has AdS_5 with homogeneous 4-form field B (so that its curvature $H = dB$ is proportional to the volume form with a constant coefficient). Brane production in that context has been studied in [12] (see also [13]). The study essentially reduces to the study of minimal charged surfaces in AdS with the homogeneous field. These surfaces were (up to minor subtleties) described in [12]. They are classified into three classes: undercharged ones (saturating a sort of BPS inequality between charge and tension of the brane), overcharged ones (breaking the BPS inequality) and BPS ones (see fig.1-4). Actually, only overcharged ones were in [12] given a tunneling interpretation, the others have infinite volume. Our innovation compared to [12] is to include junctions of those surfaces (junctions in in tunneling were first encountered in [15], in [14] junctions were extensively used in description of the induced brane pair production). Our Big Bang bounce is glued out of three peaces of branes - of BPS one, of undercharged one and of overcharged one (see fig.5). The BPS brane plays the role of RS-brane, the others are R-branes. Notice that the overcharged branes inherit the metric of the sphere, the undercharged branes - the one of AdS, and the BPS brane - the flat one, we

shall discuss this point in more details later.

The paper is organized as follows. First we describe the charged minimal surfaces in AdS with external field. Then we construct the Big Bang bounce describing tunneling into the Brane World. Finally we present our conclusions. Throughout the paper we use the test brane approximation - the same approximation as in [12] - that is we neglect the brane back reaction on the gravity and field. The construction is straightforwardly generalizable beyond this approximation along the lines of [11].

2. In this section we describe, essentially following [12], "Euclidean" minimal charged surfaces, that is, solutions of test brane worldsheet equations relevant for tunneling (the ideas of that kind of consideration of tunneling were developed in [16]). Effective action of the test brane can be taken in the following form:

$$S = T \oint \sqrt{g} + Q \oint B \tag{1}$$

where T stands for tension of the brane, Q stands for its charge, g is induced metric on the brane and B is a $(d-1)$-form field. The integrals are over brane worldsheet. If one takes the metric of AdS as in [12],

$$ds^2 = R_{ads}^2 \left(\cosh^2 \rho \, d\tau^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_{d-2}^2 \right), \tag{2}$$

where R_{ads} is the anti-de Sitter radius, and assumes that the curvature form $H = dB$ of the B-field is proportional to the volume form with a constant coefficient (flux density), and also assumes the spherical symmetry of the brane worldsheet, one reduces Eq.(1) as follows [12]:

$$S = T R_{ads}^{d-1} \Omega_{d-2} \int d\tau \left[\sinh^{d-2} \rho \sqrt{\cosh^2 \rho + \left(\frac{d\rho}{d\tau} \right)^2} - q \sinh^{d-1} \rho \right] \tag{3}$$

where $\Omega_{d-2} = \frac{2\pi^{\frac{(d-1)}{2}}}{\Gamma(\frac{d-1}{2})}$ stands for the volume of a unit d-2 sphere and q is a constant made out of the flux density, brane charge and the brane tension. $q = 1$ is identified in [12] as the BPS case. The branes with $q < 1$ will be referred to as undercharged ones and those with $q > 1$ - as overcharged.

Before describing them we would like to relate the metric Eq.(2) used in [12] to more canonical ones. Upon the change of variables,

$$\tanh \tau = \tan \theta \tag{4}$$

the metric Eq.(2) is put to the form

$$ds^2 = R_{ads}^2 \frac{d\tau^2 + d\theta^2 + \sin^2 \theta \, d\Omega_{d-2}^2}{\cos^2 \theta} \tag{5}$$

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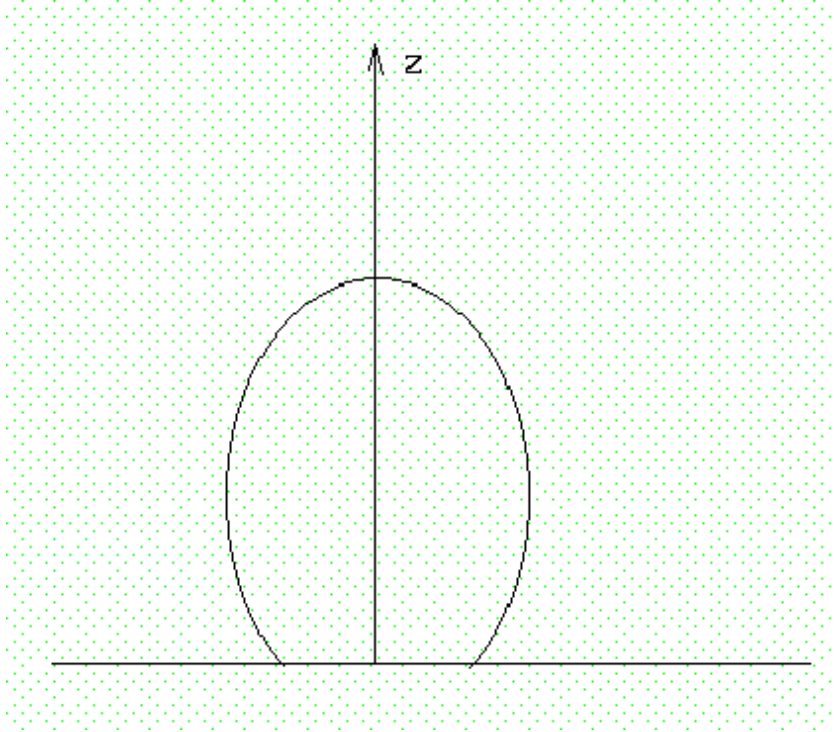Figure 1: Undercharged brane.

Finally, upon the change of variables

$$z = e^\tau \cos\theta, \ r = e^\tau \sin\theta \tag{6}$$

one obtains the following canonical form of AdS metric

$$ds^2 = R_{ads}^2 \frac{dz^2 + dr^2 + r^2 \, d\Omega_{d-2}^2}{z^2}. \tag{7}$$

Let us first consider minimal surfaces for the case with $q < 1$ which look as follows (see also fig.1):

$$\cosh\rho = \frac{\sinh\tau_m}{\sinh(\tau + a)} \tag{8}$$

where $\tanh\tau_m = q$.

Notice that we have added a modulus a compared to Eq.(4.8) in [12]. In the context of [12] this modulus represents the time translation symmetry and it is more or less irrelevant. In terms of metric Eq.(7) this modulus rather defines a scale. The moduli are necessary to describe the correct limit to the BPS case (a is sent to infinity when $q \to 1$) and they are also necessary to construct the Big Bang bounce with junctions which we are describing below. Notice also that in our construction below it is one of the coordinates
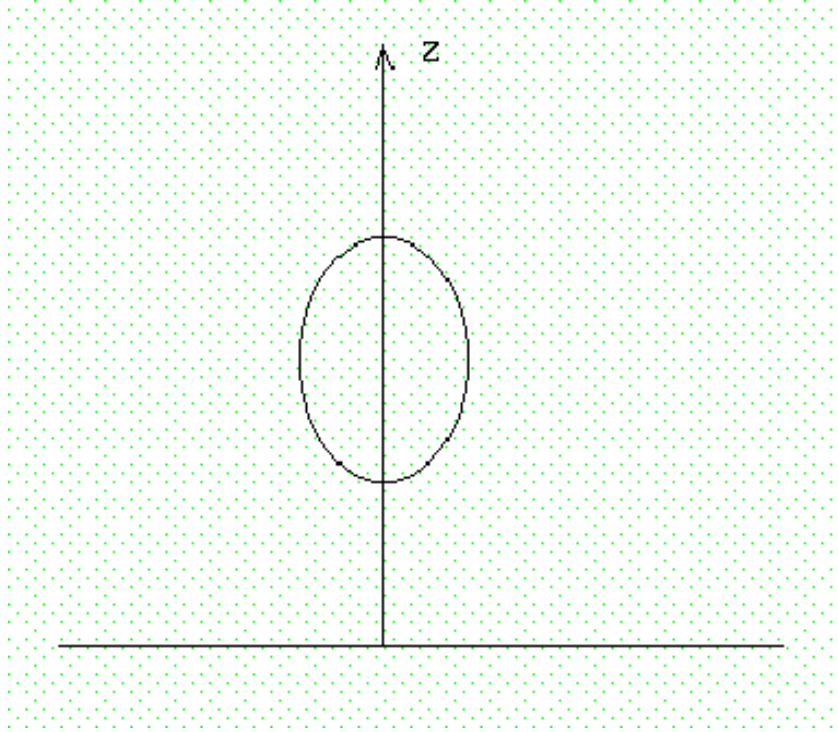
Figure 2: Overcharged brane.

orthogonal to z which is interpreted as Euclidean time, unlike [12], where the role of Euclidean time is played by τ.

In coordinates Eq(4) the undercharged surfaces take the form

$$\cos\theta = \frac{\sinh(\tau + a)}{\sinh\tau_m}. \tag{9}$$

As we pointed in the Introduction, restriction of the AdS_5 metric Eq.(5) onto the undercharged surfaces Eq.(11) gives the metric of AdS_4.

Let us now turn to the case with $q > 1$. The relevant charged minimal surfaces look as follows (see also fig.2):

$$\cosh\rho = \frac{\cosh\rho_m}{\cosh(\tau + b)} \tag{10}$$

where $\tanh\rho_m = 1/q$. We have again added a modulus b compared to Eq.(4.5) in [12]. In coordinates Eq.(4) the overcharged surfaces take the form

$$\cos\theta = \frac{\cosh(\tau + b)}{\cosh\rho_m}. \tag{11}$$

As we pointed in the Introduction, restriction of the AdS_5 metric Eq.(5) onto the overcharged surfaces Eq.(11) gives the metric of the sphere S_4.
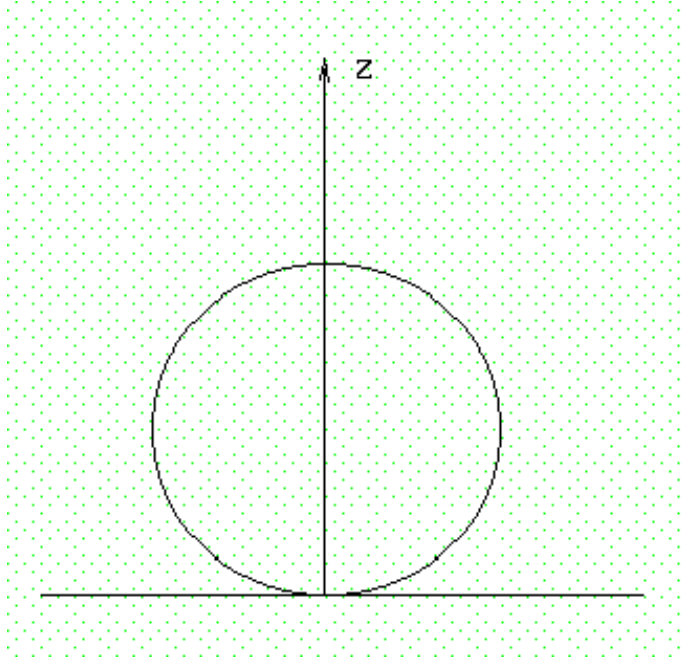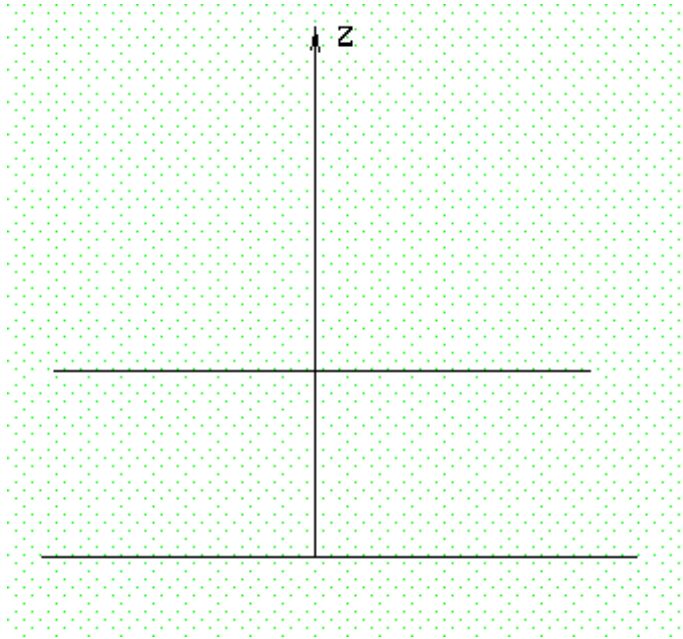
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Figure 3: BPS brane.

Figure 4: BPS brane. It is obtained from the previous case by inversion.

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The BPS case (q=1) can be obtained as a limit from either of the cases above. Corresponding surfaces look as follows (see also fig.3):

$$\cos\theta = \frac{1}{z_0}e^\tau \tag{12}$$

where z_0 is a constant. Upon the inversion transformation one obtains

$$e^\tau \cos\theta = z_0 \tag{13}$$

In terms of Eq.(7) these are the surfaces $z = z_0$ (fig.4). Apparently, restriction of the AdS metric onto these surfaces gives flat Euclidean metric.

Only overcharged surfaces admit a tunneling interpretation [12], since undercharged and BPS ones reach the boundary of AdS space and thus have infinite volume and infinite effective action.

3. We shall now construct the bounce which describes tunneling into the Brane World. It is glued out of three pieces - a piece of BPS brane located along $z = z_0$ section, Eq.(13), and playing a role of RS-brane in the Brane World, a piece of undercharged brane, Eq.(9), located above the BPS brane (in the fixed coordinates of the type of Eq.(7) and playing a role of one R-brane, and a piece of overcharged brane, Eq.(11), located below the BPS brane and playing the role of the other R-brane (see fig.5). All three pieces are glued along the junction manifold. The usual junction conditions are the charge conservation and the tension forces balance at the junction [17] . We shall assume that there is no junction energy contribution to the effective action (such contributions were studied in [18] in the context of specific central charges), though this assumption is not crucial for the existence of solution.

Apparently, the configuration sketched above has a finite effective action since none of the constituting pieces reaches the AdS boundary , hence the tunneling goes with a finite probability which can be easily computed. Notice that analogously to constructions in [12] one needs a brane breaking BPS inequality in order to have a finite probability of tunneling. An example of this type of brane in string theory was given in [12].

Junction manifold is of the type $\theta = const$, $\tau = const$, and since overall rescaling of the solution is not important (the effective action is invariant under total rescaling, or, equivalently, under total shift in τ-coordinate), we take it as follows:

$$\theta = \theta_c, \ \tau_c = 0 \tag{14}$$

From Eqs.(13),(9),(11) one immediately obtains

$$\cos\theta_c = z_0,$$
$$\sinh a = \sinh\tau_m \cos\theta_c \tag{15}$$
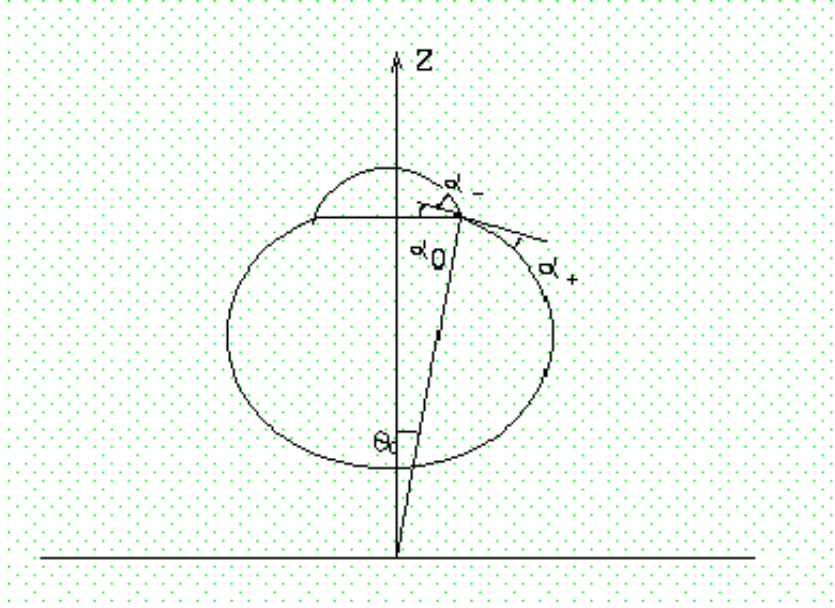$$\cosh b = \cosh\rho_m \cos\theta_c.$$

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Figure 5: Big Bang bounce, or tunneling from AdS to the Brane World.

The Eqs.(13),(9),(11), (14),(15) specify the geometry of the Big Bang bounce. However we still have to define charges of the branes involved and to verify that the junction conditions are fulfilled.

The charge conservation, with appropriate choice of orientation of the branes, reads

$$Q_0 = Q_- - Q_+. \tag{16}$$

Hereafter subscripts "0","-", and "+" indicate the BPS brane, the undercharged brane and the overcharged brane. All Q's are assumed to be positive.

The force balance condition obviously reads (we take projections onto $\partial/\partial\theta$ and onto $\partial/\partial\tau$ directions):

$$T_0 \cos \alpha_0 + T_- \cos \alpha_- = T_+ \cos \alpha_+$$
$$T_0 \sin \alpha_0 + T_+ \sin \alpha_+ = T_- \sin \alpha_- \tag{17}$$

where the angles are defined on fig.5. From geometry of the picture and using Eqs.(11),(9),(14), (15) one obtains

$$\alpha_0 = \theta_c$$
$$\tan \alpha_- = \frac{\sin \theta_c \sinh \tau_m}{\cosh a} \tag{18}$$
$$\tan \alpha_+ = \frac{\sin \theta_c \cosh \rho_m}{\sinh b}$$

According to the above definition of q (see Eq.(3)) we take the following

parametrization of the tensions of the three pieces of the bounce:

$$T_0 = Q_0 T, \ T_- = \frac{Q_-}{q_-} T, \ T_+ = \frac{Q_+}{q_+} T \qquad (19)$$

where $q_- = \tanh \tau_m$, $q_+ = 1/\tanh \rho_m$.

Substituting Eq.(19) into the force balance condition Eq.(17) and using the charge conservation condition Eq.(16) one obtains a linear system for the charges of R-branes:

$$Q_- \left(\cos \theta_c + \frac{\cos \alpha_-}{q_-} \right) - Q_+ \left(\cos \theta_c + \frac{\cos \alpha_+}{q_+} \right) = 0$$

$$Q_- \left(\sin \theta_c - \frac{\sin \alpha_-}{q_-} \right) - Q_+ \left(\sin \theta_c - \frac{\sin \alpha_+}{q_+} \right) = 0 \qquad (20)$$

Using Eqs.(15,(18) one can straightforwardly verify that determinant of this linear system is equal to zero, so the system is compatible and defines the ratio of charges of the R-branes at which the force balance condition Eq.(17) is satisfied. This completes our construction of the Big Bang bounce.

4. We have described spontaneous creation of the brane configuration which looks very similar to those involved in the so called Brane World scenario. The initial background consists of AdS_5 metric and homogeneous 4-form field (so that its curvature form is proportional to the volume form with a constant coefficient), the final state (in "Minkowski" picture) involves a spatially finite, flat piece of brane at rest (RS-brane) restricted by the junction with two expanding R-branes, the distance between RS- and R-branes and the size of the R-branes rapidly increasing. Since RS-brane is located along the same section of AdS_5 as the physical brane in [2], we take for granted the 4d localization of gravity and the validity of 4d Newton law (modulo decreasing finite size effects) on the RS-brane. We stress that no negative tension branes are involved in our picture.

To conclude, we would like to point out following interesting features of this kind of model.

i)As we already pointed out in [11], in a model of this kind the early Universe is five dimensional so one could expect interesting cosmological implications.

ii)Since the RS-brane is a piece of the BPS brane, we expect that 4d supersymmetry is broken by the finite size effects. Moreover the scale of the SUSY breaking effects changes in time.

iii)Interesting cosmological consequences should follow the fact that a matter, which is assumed to be localized on the branes, can penetrate via junction manifold to/from our 4d Universe from/to the RS branes (junctions in the Brane World context were considered in [19]). In particular there are models with chirality localized on the junctions so in such case we would have a kind of anomaly phenomena.

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iv) Finally, note that string or branes connecting RS- and R-branes would result in the particle or extended objects with increasing masses (tensions) in the Brane World.

References

[1] I. Antoniadis, S. Dimopoulos and G. Dvali, Nucl. Phys. **B516** (1998) 70 [hep-th/9710204]

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].

[3] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B125** (1983) 136
K. Akama, Lecture Notes in Physics, 176, (Springer-Verlag, 1983), 267-271, hep-th/0001113.

[4] J. Polchinski, Phys. Rev. Lett. **75** (1995), 4724
E. Witten, Nucl. Phys. **B460** (1996) 335

[5] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, hep-th/0002072.

[6] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, hep-ph/9912552.
R. Gregory, V. A. Rubakov and S. M. Sibiryakov, hep-th/0003109.
I. I. Kogan and G. G. Ross, hep-th/0003074.
G. Dvali, G. Gabadadze and M. Porrati, hep-th/0002190
C. Csaki, J. Erlich, T. J. Hollowood and Y. Shirman, hep-th/0001033
C. Csaki, J. Erlich, T. J. Hollowood and J. Terning, hep-th/0003076
C. Csaki, J. Erlich and T. J. Hollowood, hep-th/0002161, hep-th/0003020,
L. Pilo, R. Rattazzi and A. Zaffaroni, hep-th/0004028.
Y. S. Myung and G. Kang, hep-th/0005206.

[7] E. Witten, hep-ph/000229.
G. Dvali, G. Gabadadze and M. Porrati, hep-th/0003054

[8] G. Dvali, G. Gabadadze and M. Porrati, hep-th/0005016

[9] J. D. Brown and C. Teitelboim, Nucl. Phys. **B297**, 787 (1988).

[10] R. Bousso and J. Polchinski, hep-th/0004134
J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, hep-th/0005276

[11] A. Gorsky and K. Selivanov, hep-th/0005066, to be published in Phys.Lett.B

[12] J. Maldacena, J. Michelson and A. Strominger, JHEP **9902**, 011 (1999) [hep-th/9812073].

[13] M. Cvetic, S. Griffies and H. H. Soleng, Phys. Rev. **D48**, 2613 (1993) [gr-qc/9306005]
M. Cvetic and J. Wang, Phys. Rev. **D61**, 124020 (2000) [hep-th/9912187].

[14] A. Gorsky and K. Selivanov, Nucl. Phys. **B571**, 120 (2000) [hep-th/9904021]

[15] K. G. Selivanov and M. B. Voloshin, JETP Lett. **42**, 422 (1985).

[16] I. Y. Kobzarev, L. B. Okun and M. B. Voloshin, Sov. J. Nucl. Phys. **20**, 644 (1975).

[17] J. Schwarz, Nucl. Phys. (Proc. Suppl) **B55** (1997)

[18] G. W. Gibbons and P. K. Townsend, Phys. Rev. Lett. **83**, 1727 (1999) [hep-th/9905196].
S. M. Carroll, S. Hellerman and M. Trodden, Phys. Rev. **D61**, 065001 (2000)
A. Gorsky and M. Shifman, Phys. Rev. **D61**, 085001 (2000) [hep-th/9909015].

[19] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000) [hep-th/9907209].
C. Csaki and Y. Shirman, Phys. Rev. **D61**, 024008 (2000) [hep-th/9908186].
S. M. Carroll, S. Hellerman and M. Trodden, hep-th/9911083.
S. Nam, JHEP **0003**, 005 (2000) [hep-th/9911104]
T. Nihei, hep-th/0005014